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HOWARD UNIVERSITY

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The Dynamics and Control of the In-Orbit SCOLE Configuration - NASA-NSG 1414

ABSTRACT

The study of the dynamics of the Spacecraft Control Laboratory Experiment (SCOLE)[1] is extended to emphasize the synthesis of control laws for both the linearized system as well as the large amplitude slewing maneuvers required to rapidly reorient the antenna line of sight. For control of the system through small amplitude displacements from the nominal equilibrium position LQR techniques are used to develop the control laws. Pontryagin's maximum principle is applied to minimize the time required for the slewing of a general rigid spacecraft system. The minimum slewing time is calculated based on a quasi-linearization algorithm for the resulting two point boundary value problem.[2] The effect of delay in the control input on the stability of a continuously acting controller (designed without considering the delay) is studied analytically for a second order plant. System instability can result even for delays which are only a small fraction of the natural period of motion.[3]

REFERENCES

1. Taylor, L.W. and Balakrishnan, A.V., "A Mathematical Problem and a Spacecraft Control Laboratory Experiment (SCOLE) used to Evaluate Control Laws for Flexible Spacecraft ... NASA/IEEE Design Challenge," Jan. 1984.

2. Li, Feiyue and Bainum, P.M., "Minimum Time Attitude Slewing Maneuvers of a Rigid Spacecraft," accepted for presentation, AIAA 26th Aerospace Sciences Meeting, Reno, Nevada, Jan. 11-14, 1988.

3. Reddy, A.S.S.R. and Bainum, P.M., "Stability Analysis of Large Space Structure Control Systems with Delayed Input," 6th VPI & SU/AIAA Symposium on Dynamics and Control of Large Structures, Blacksburg, Va., June 29 - July 1, 1987.

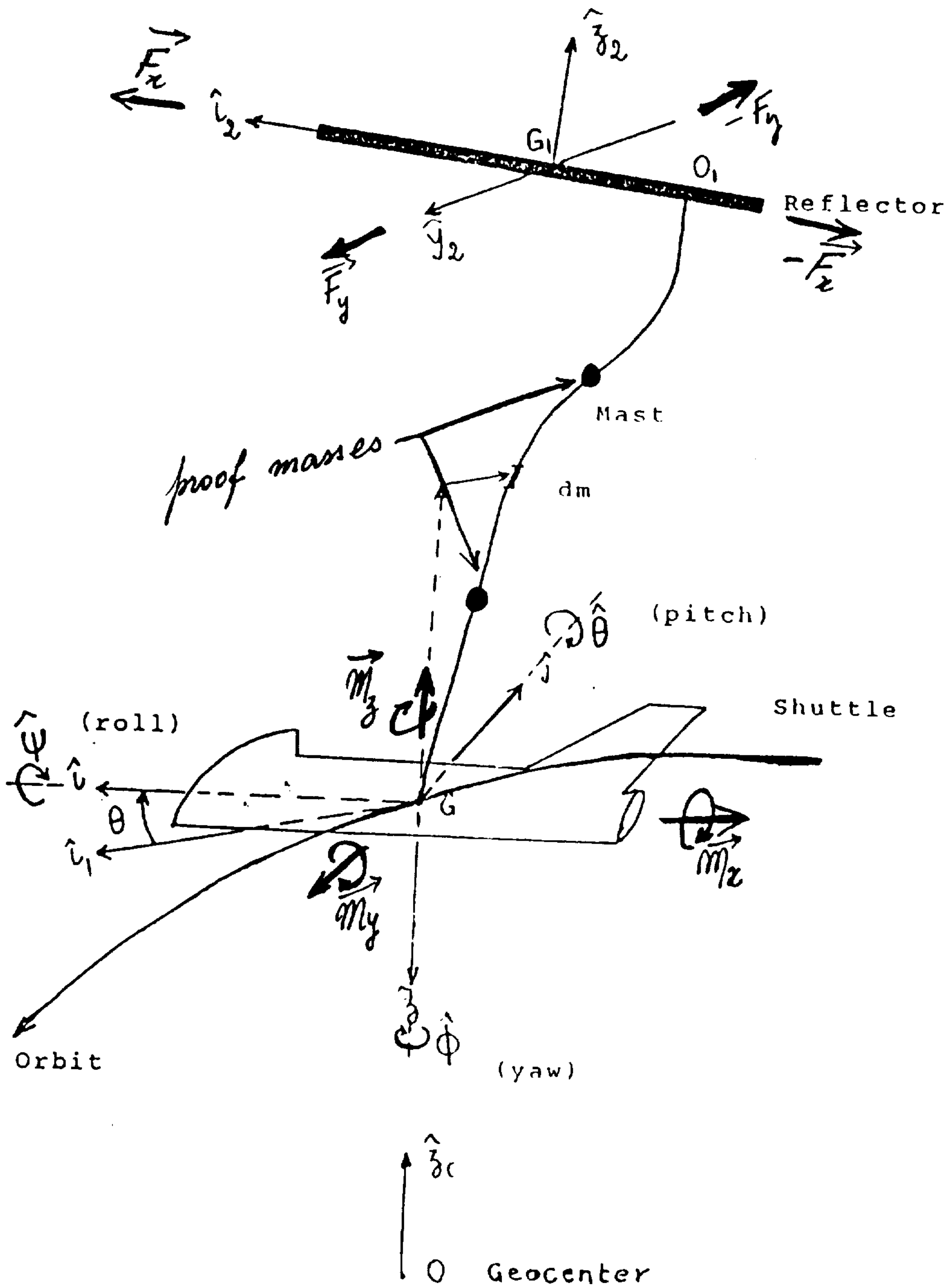


FIGURE III-1: THE 3-D GEOMETRY OF THE SCOLE CONFIGURATION IN ITS DEFORMED STATE

. Equations of Motion

· Derived using a Newton-Euler approach

. Assumptions

- Reflector and Shuttle rigid
- Mast has constant cross-section
- It is assumed to undergo small elastic deformations only
- Its modal shapes in orbit are assumed to be the same as those of an identical non-rotating beam.

Stability Analysis (Rigidized SCOLE)

A stability analysis of the rigidzed SCOLE was conducted for the following configurations:

a) Rigid - no offset. Pitch motion decouples from roll and yaw in the linear ranges. System not stable

b) Rigid - with offset parallel to roll axis. Pitch motion still decouples from roll and yaw in the linear range. System unstable.

c) Rigid - With both offsets (parallel to roll and pitch axes). The motions in all 3 degrees of freedom are coupled. System found to be unstable.

. Control Laws

Assumption: All the states of the system are available.

It was suggested by J.G. Lin that an intuitively appealing practical approach to achieve the LOS pointing objective is a two-stage procedure. (a) Slew as if rigid then, (b) damp-out flexible dynamics.

. The linear regulator theory used here to control

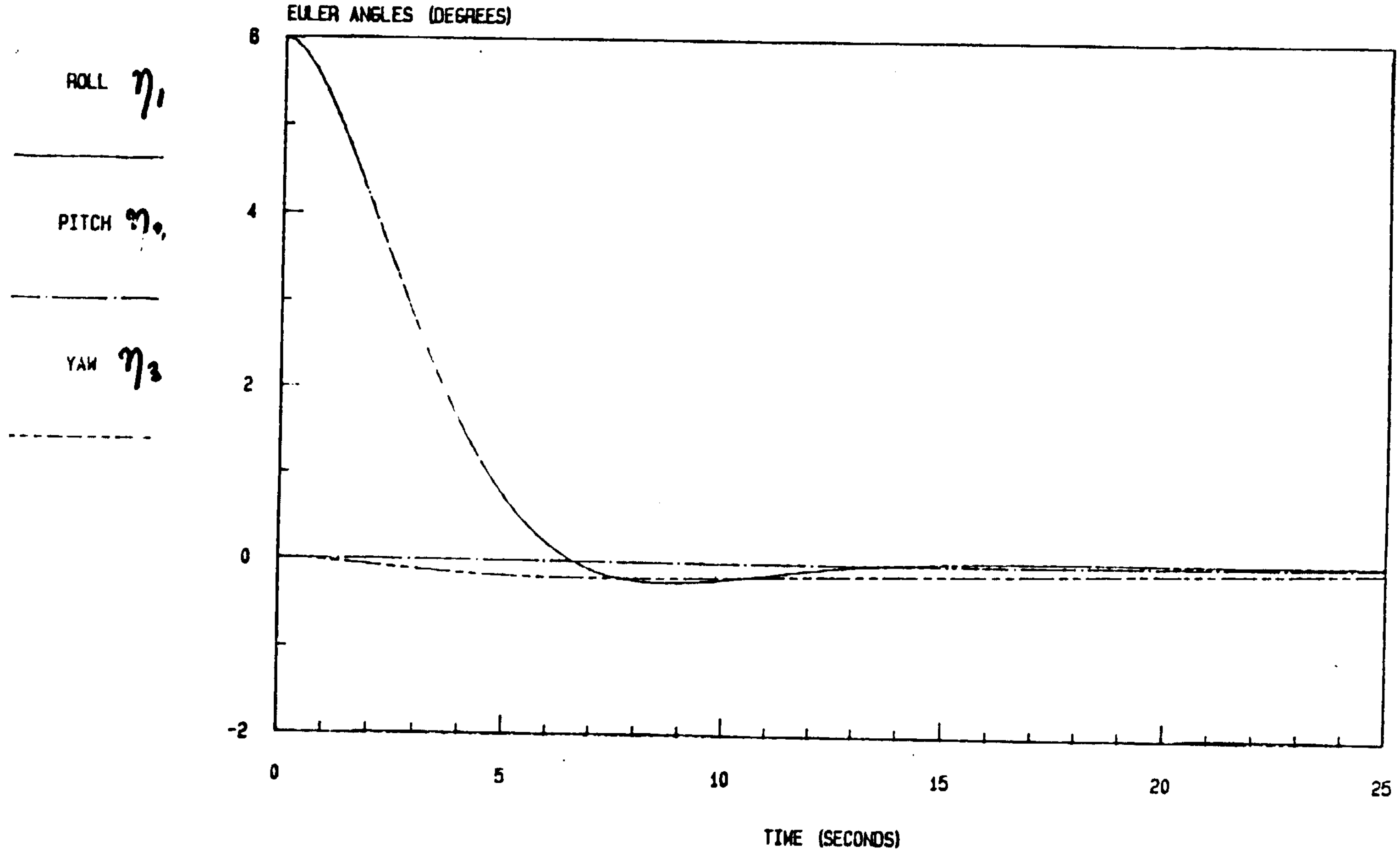
- the linear model of the rigidized SCOLE,
- The linear model of the actual SCOLE configuration including the first four flexible modes of the mast.

Next

Preliminary slew maneuvers of rigidized SCOLE.

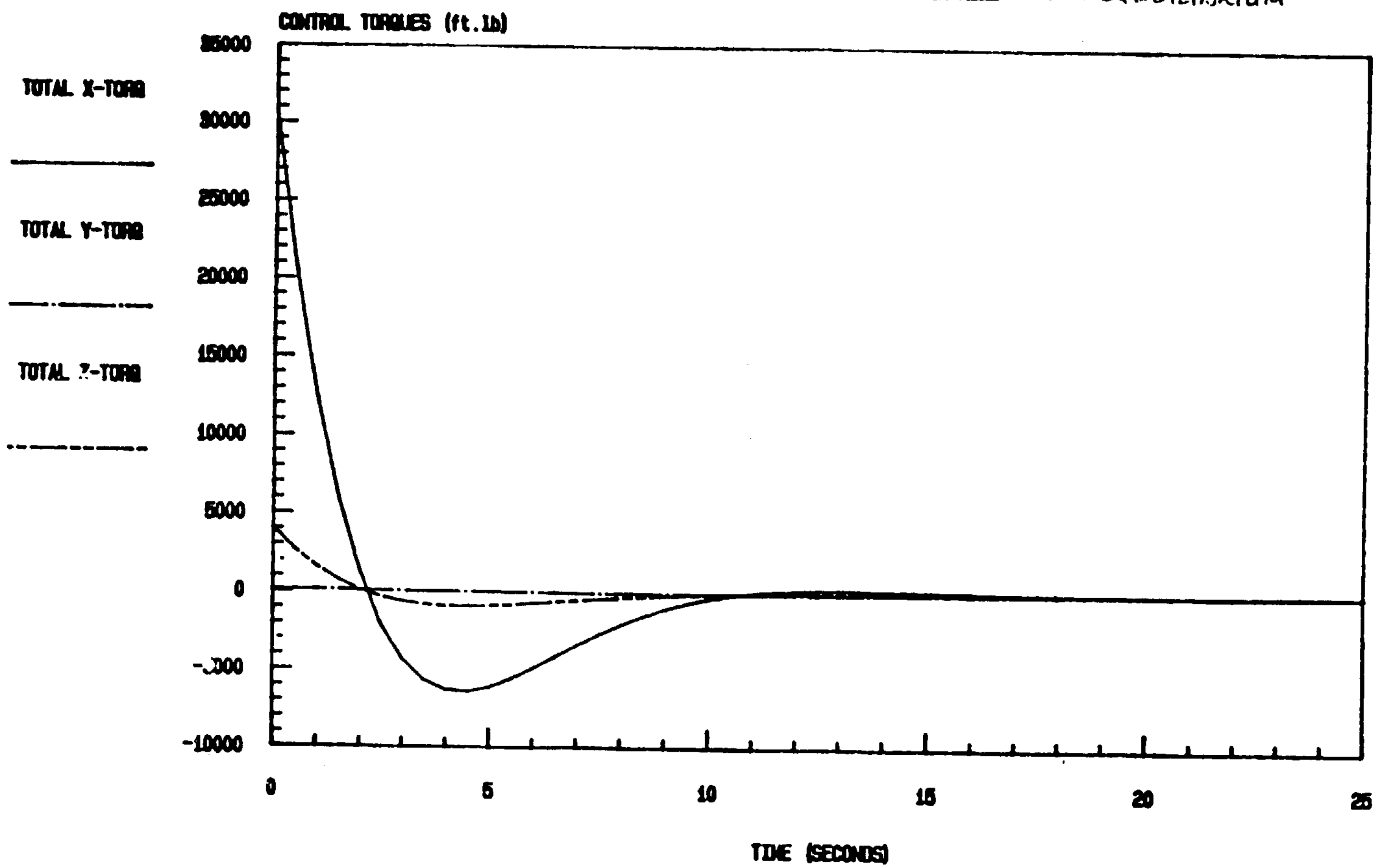
SCOLE: TRANSIENT RESPONSES

6.0 DEG. INITIAL PERTURBATION IN ROLL FROM EQUILIBRIUM



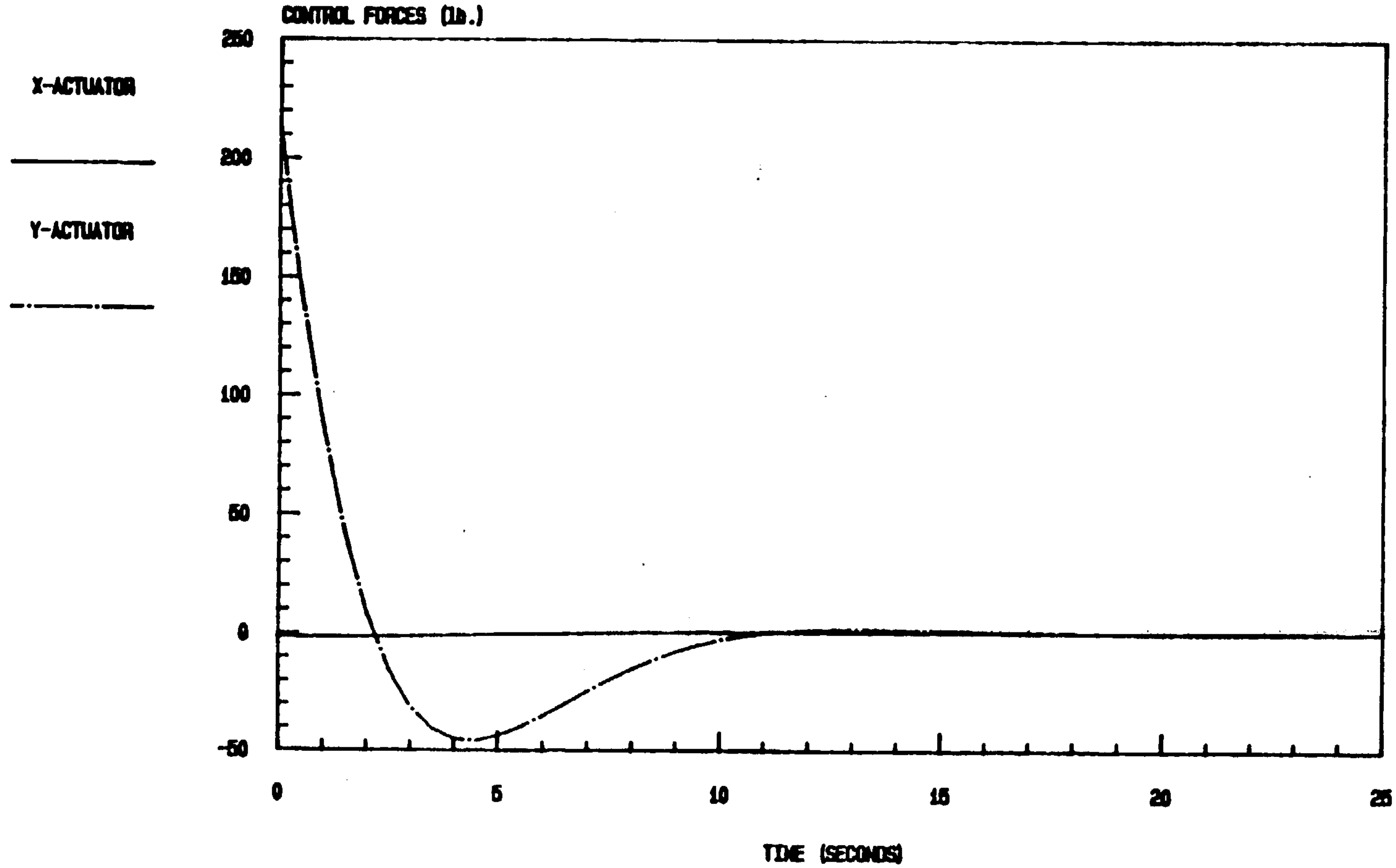
RIGID SCOLE - CONTROL EFFORTS

RESPONSE TO A 6.0 DEGREES IN ROLL FROM. EQUILIBRIUM



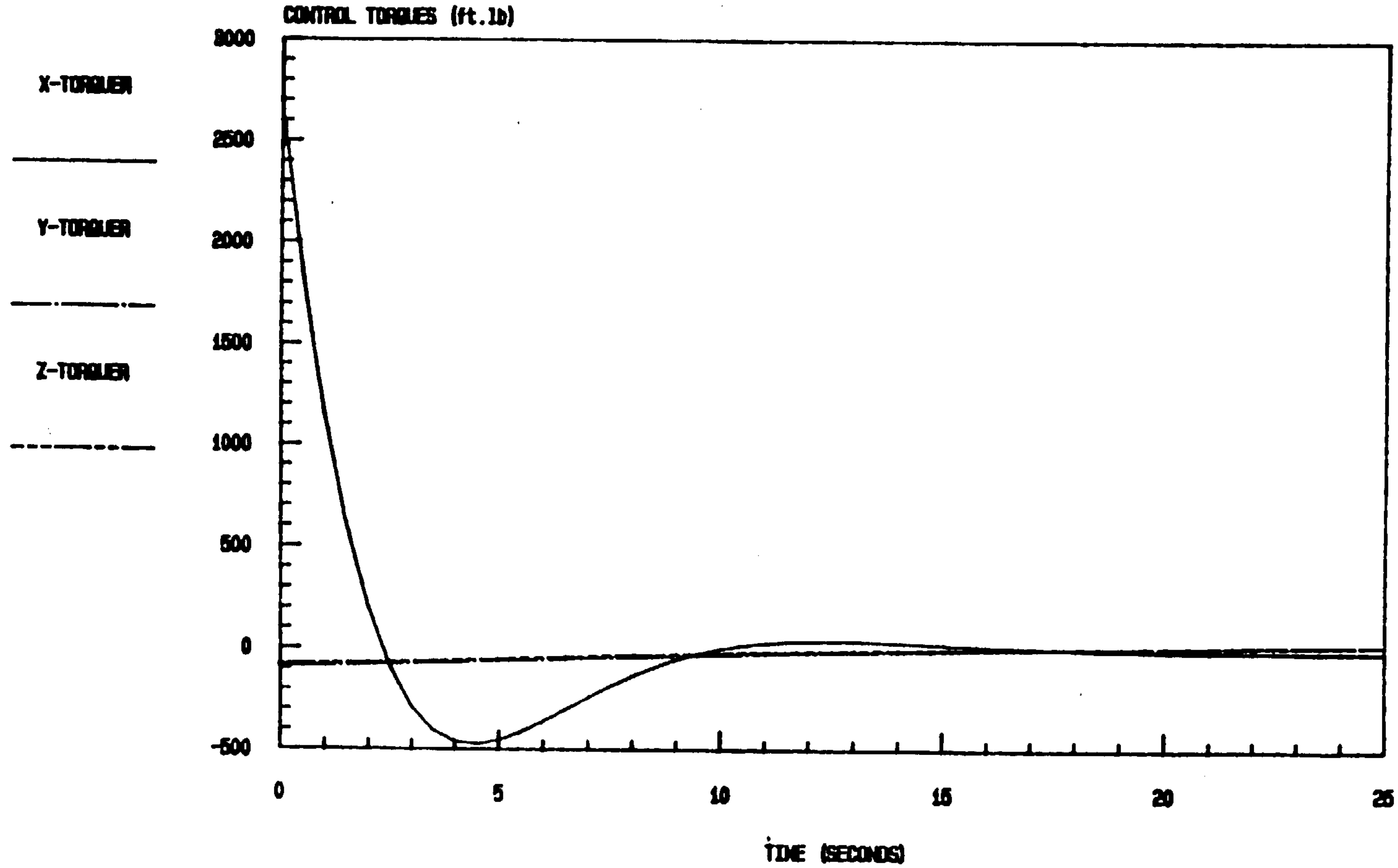
RIGID SCOLE - CONTROL EFFORTS

RESPONSE TO A 6.0 DEGREES IN ROLL FROM EQUILIBRIUM



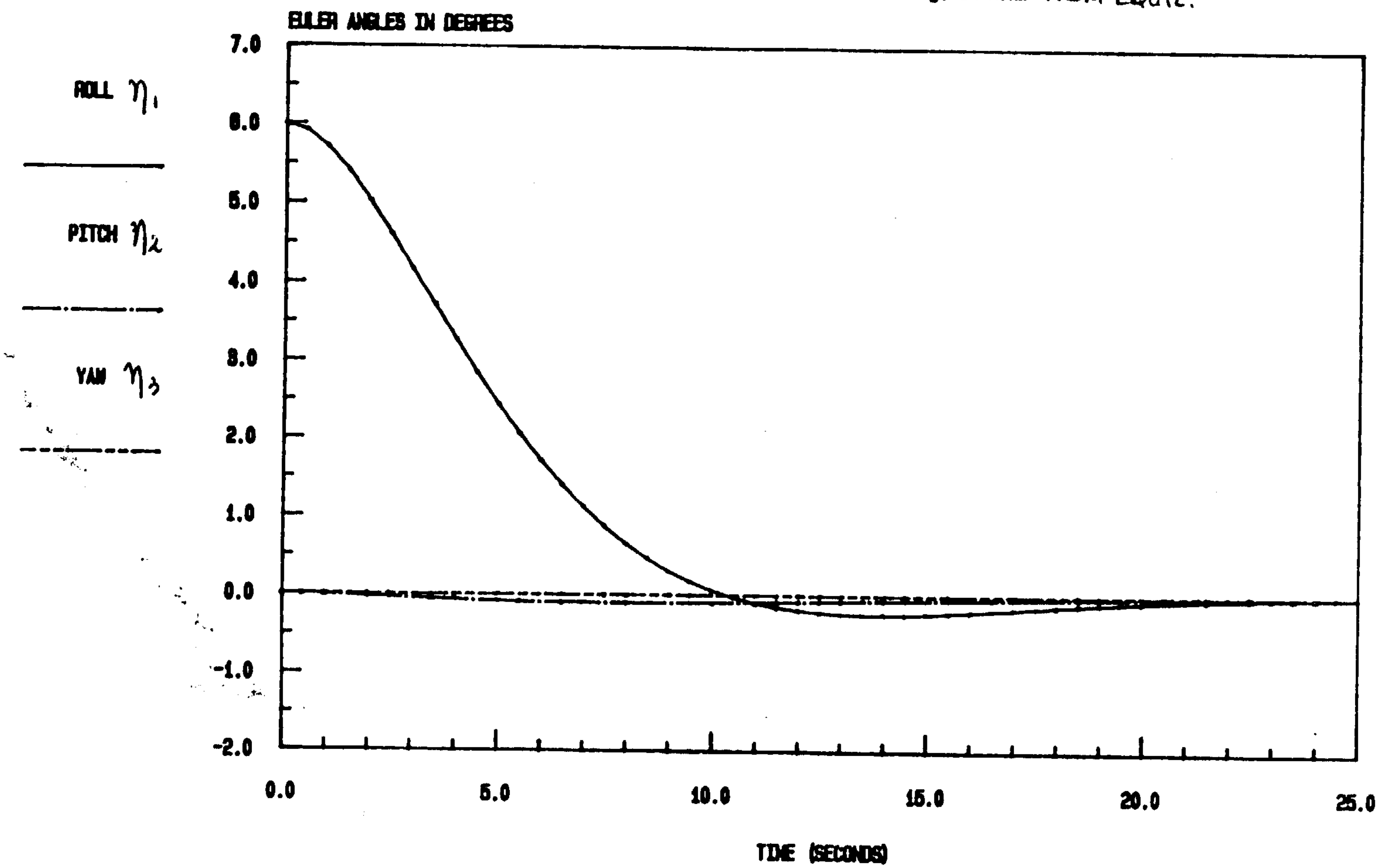
RIGID SCOLE - CONTROL EFFORTS

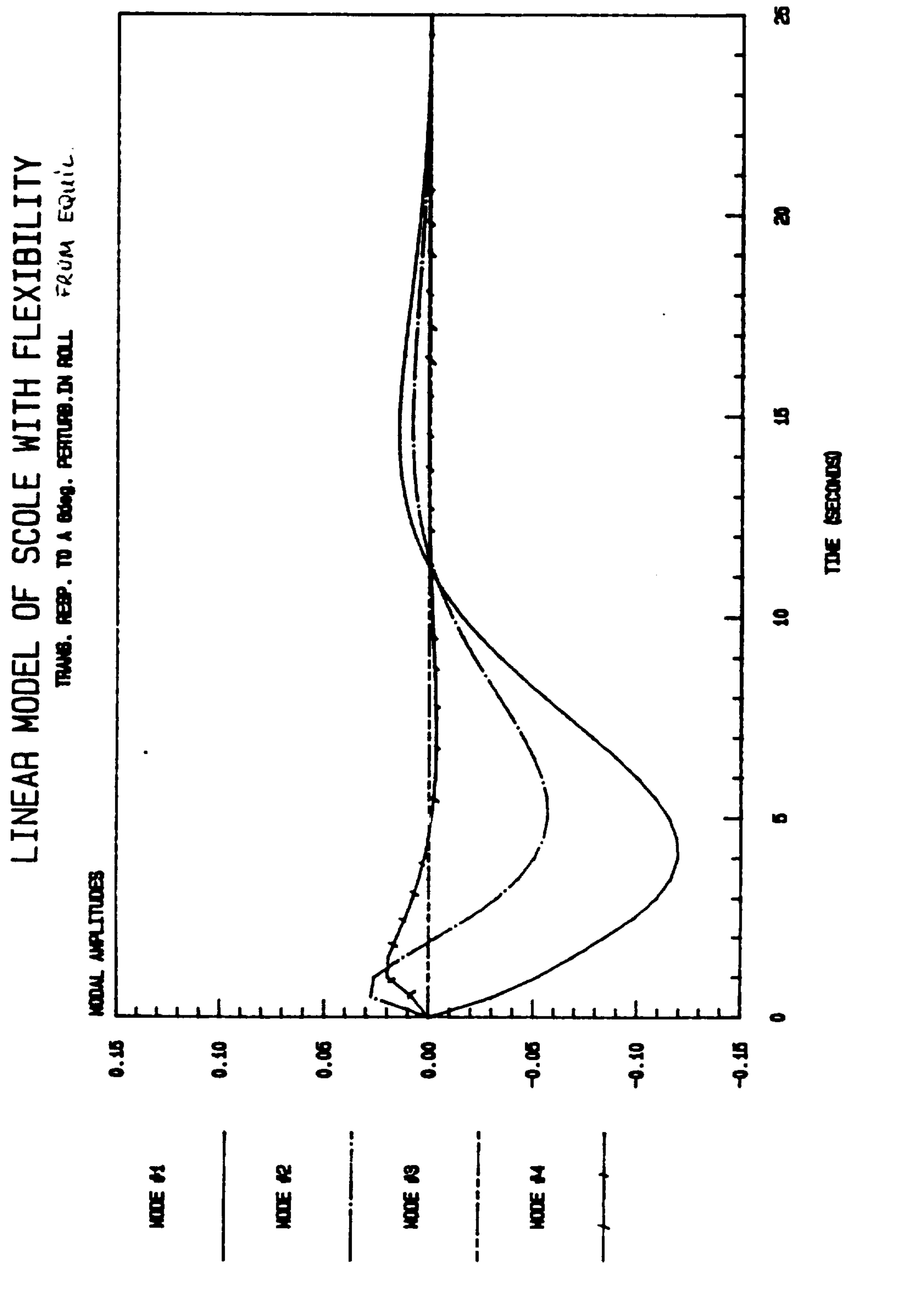
RESPONSE TO A 8.0 DEGREES IN ROLL FROM EQUILIBRIUM



LINEAR MODEL OF SCOLE WITH FLEXIBILITY

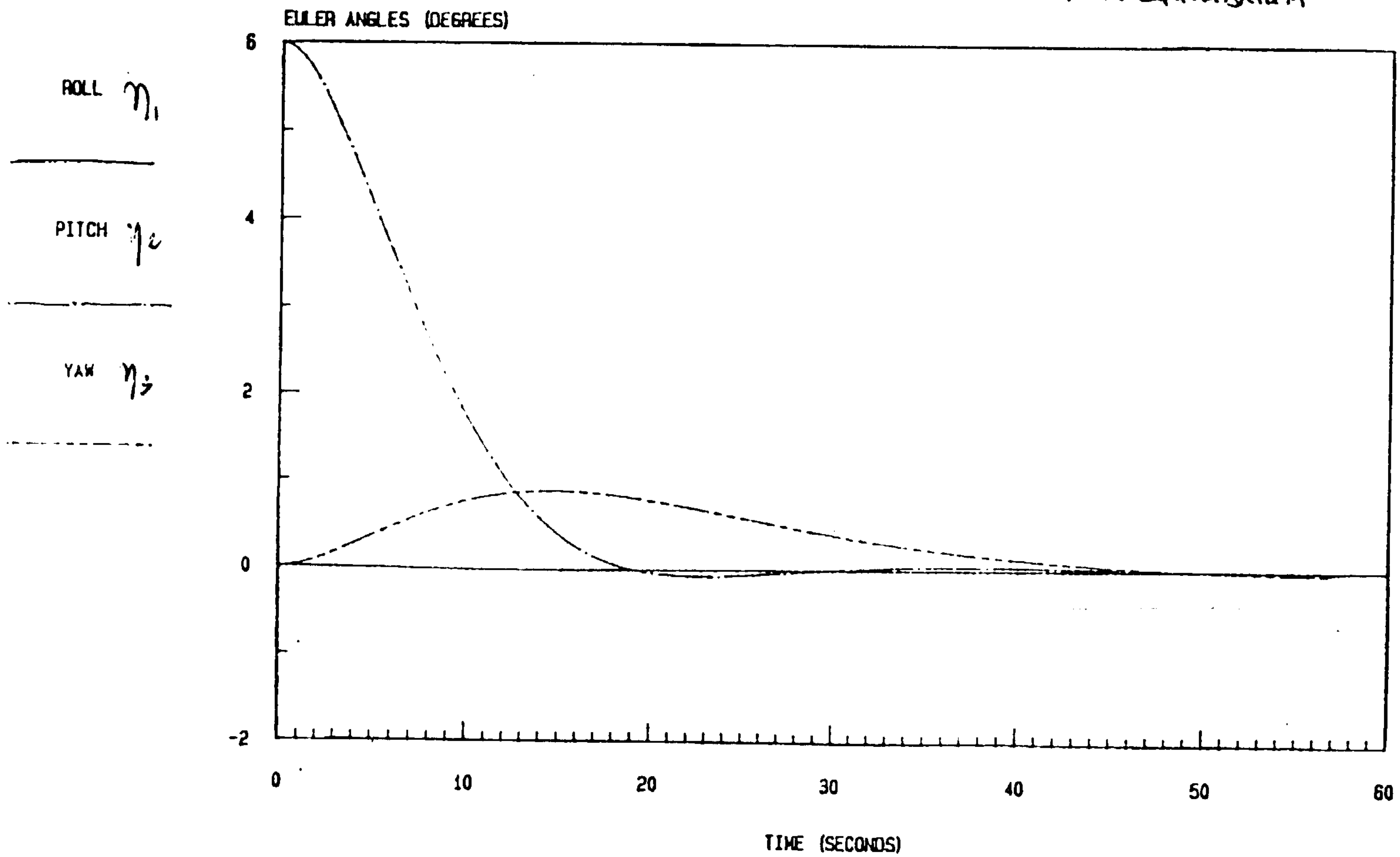
TRANS. RESP. TO AN INITIAL 6deg. IN ROLL FROM EQUIL.




LINEAR MODEL OF SCOLE WITH FLEXIBILITY
TRANS. RESP. TO A 6deg. PERTURB. IN ROLL FROM EQUIL.
MODAL AMPLITUDES
TIME (SECONDS)
MODE #1
MODE #2
MODE #3
MODE #4

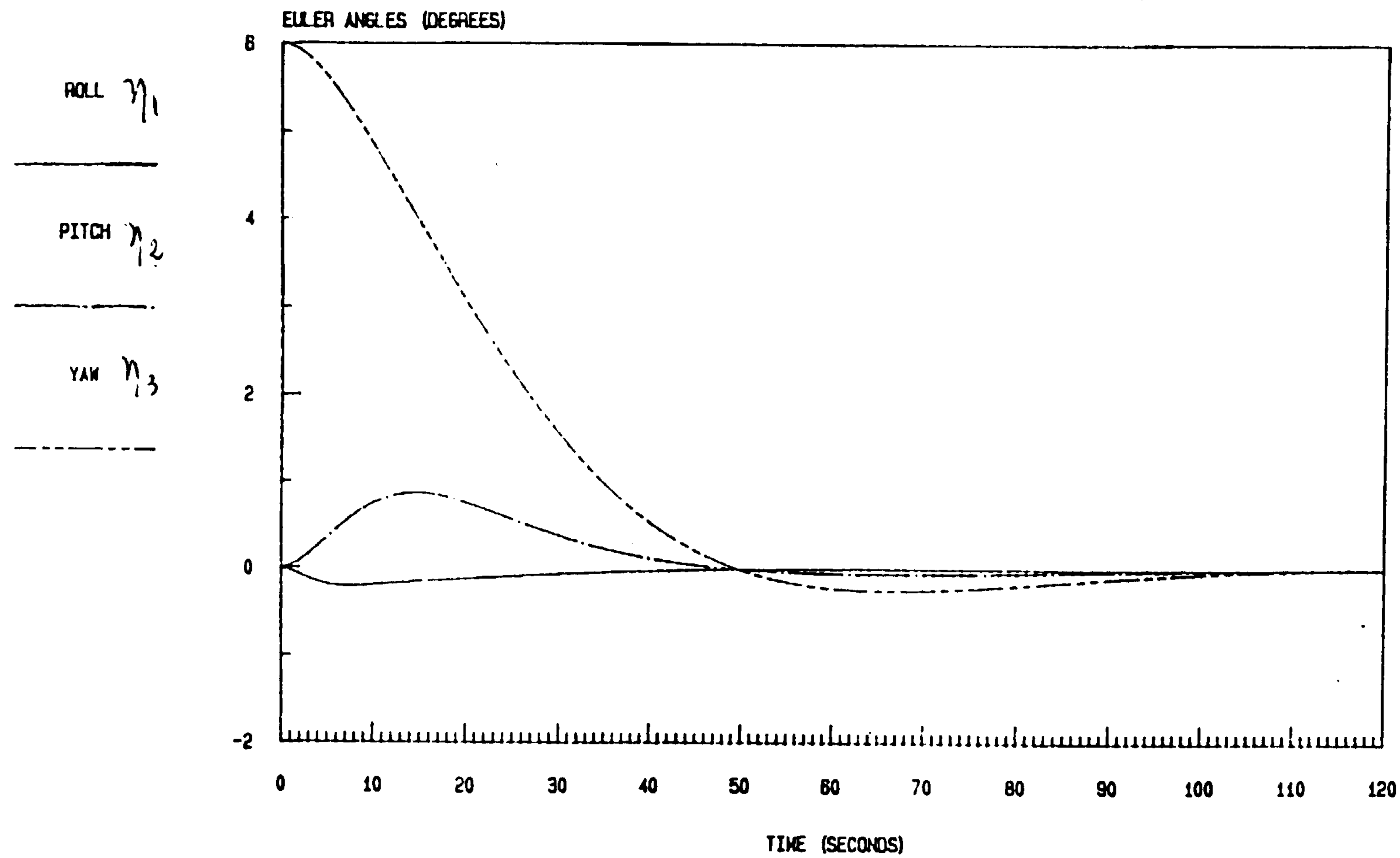
SCOLE: TRANSIENT RESPONSES

6.0 DEG. INITIAL PERTURBATION IN PITCH FROM EQUILIBRIUM



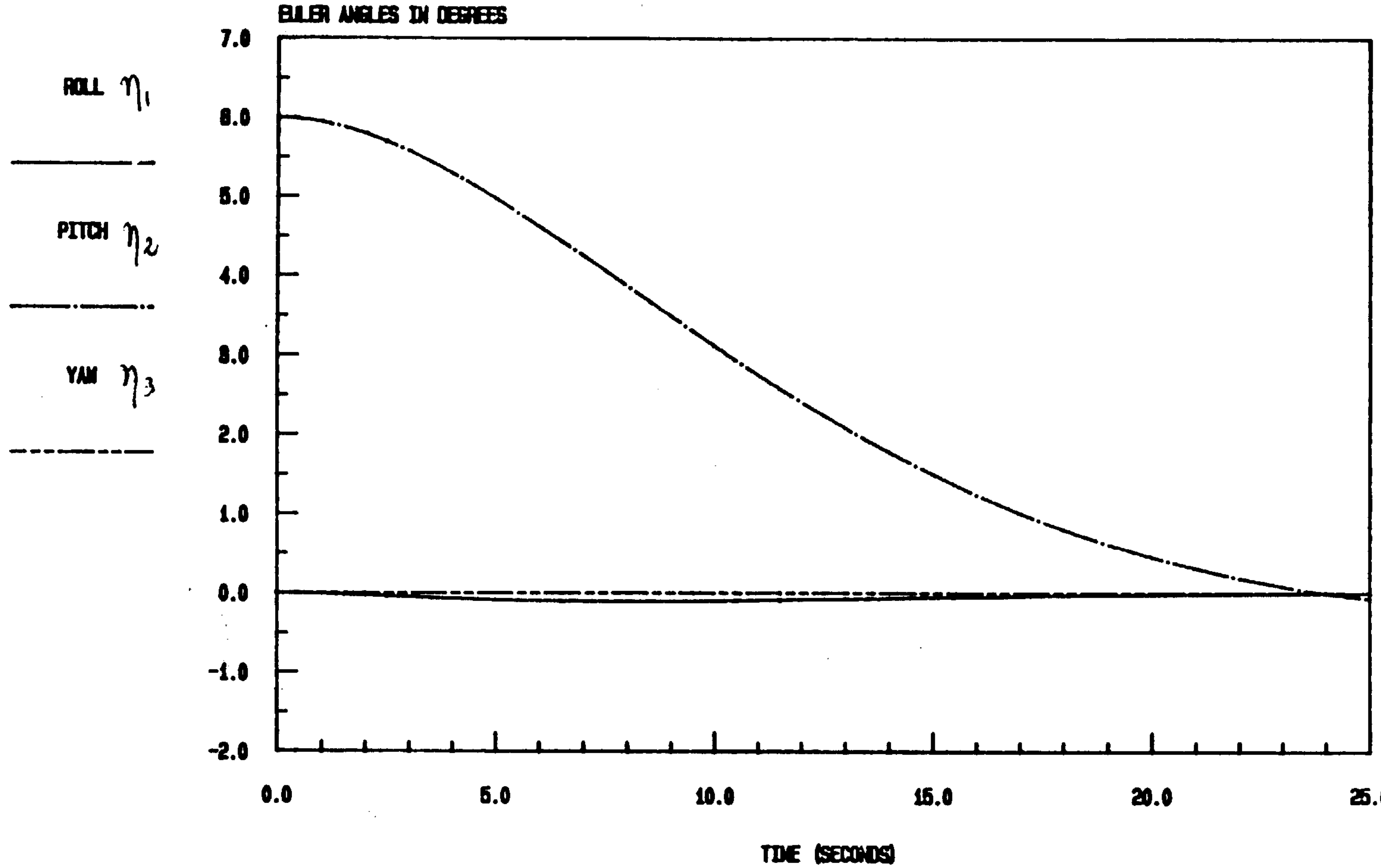
SCOLE: TRANSIENT RESPONSES

6.0 DEG. INITIAL PERTURBATION IN YAW FROM EQUILIBRIUM



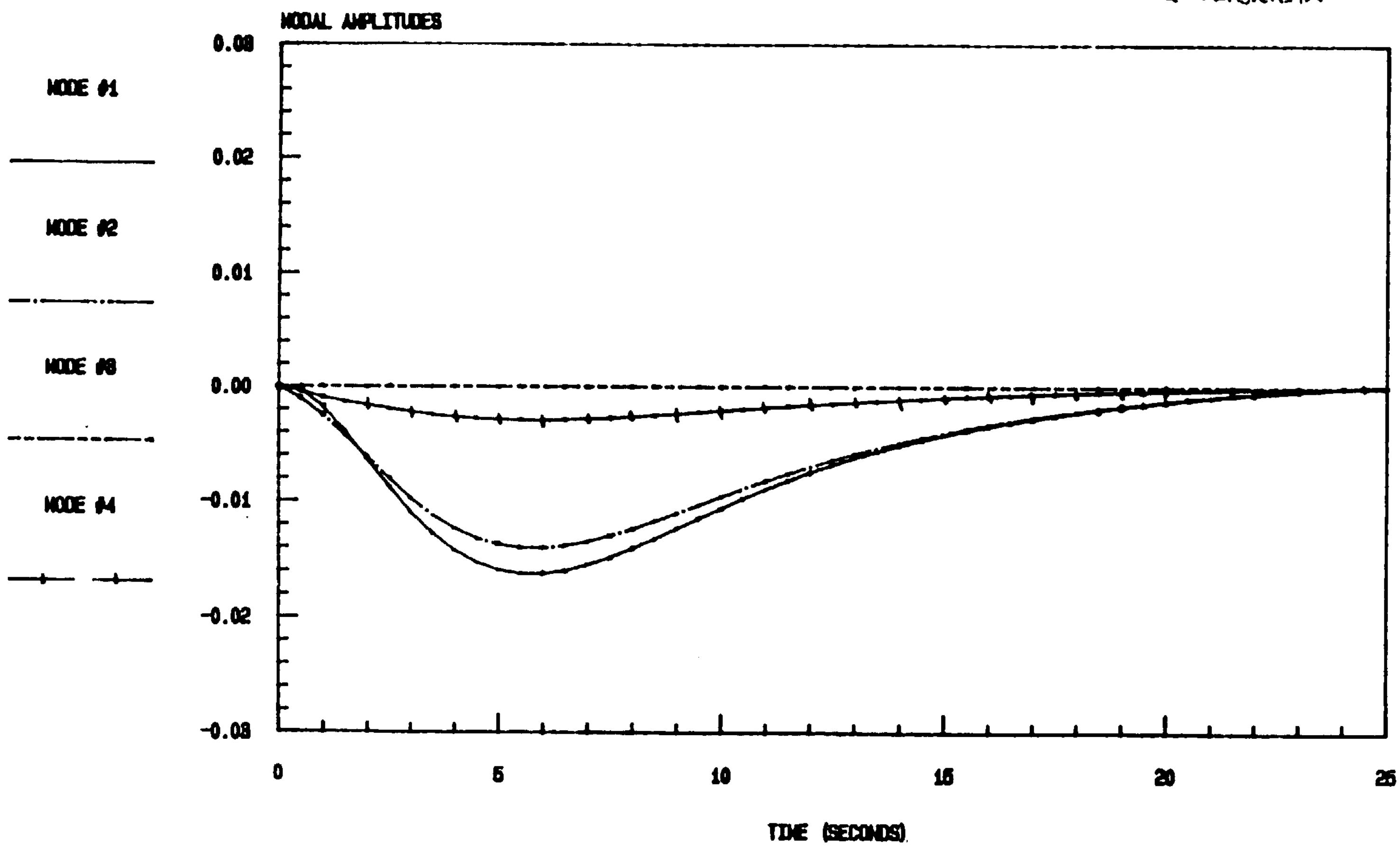
LINEAR MODEL OF SCOLE WITH FLEXIBILITY

TRANS. RESP. TO A 6deg. PERTURB.IN PITCH FROM EQUILIBRIUM



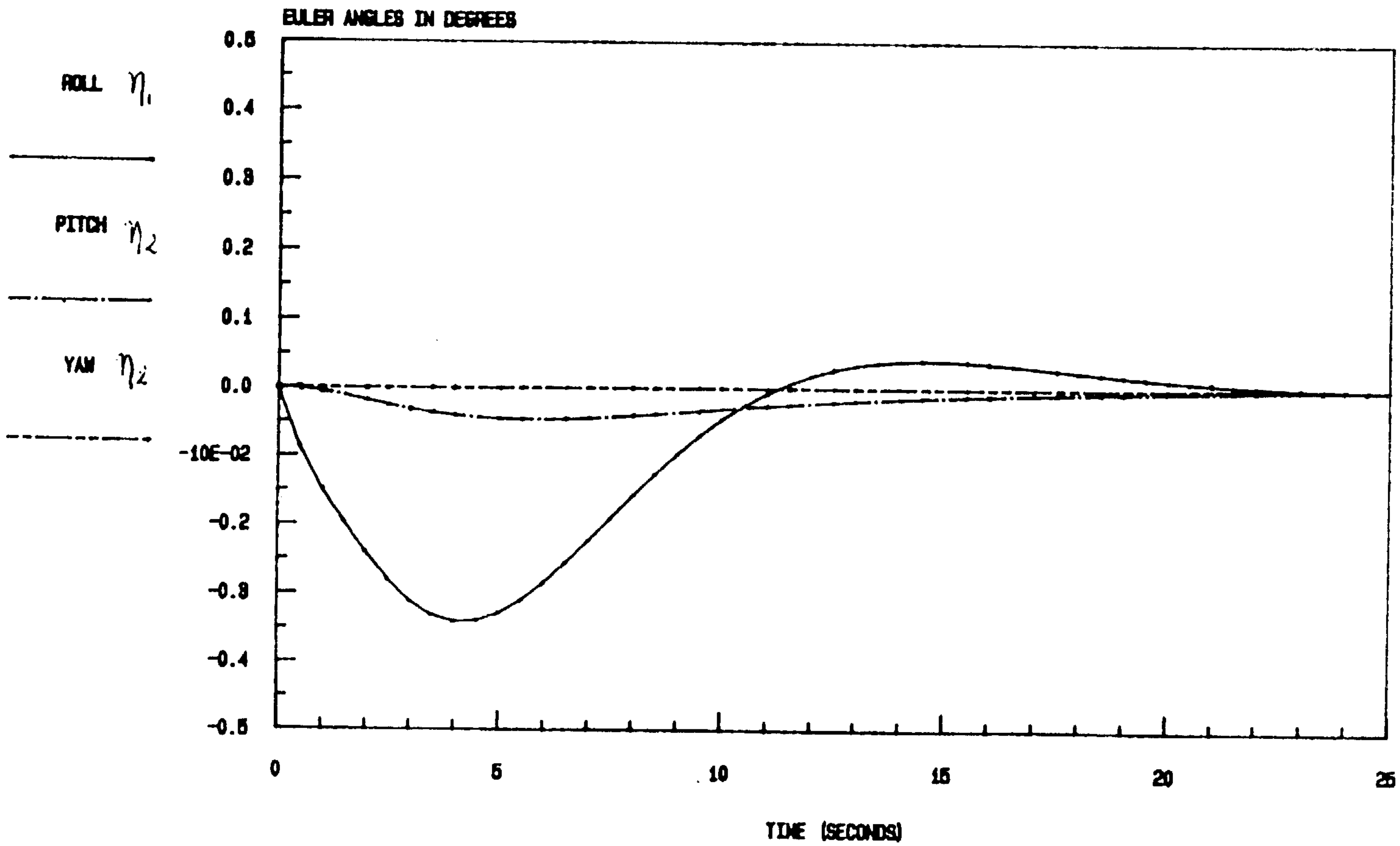
LINEAR MODEL OF SCOLE WITH FLEXIBILITY

TRANS. RESP. TO A 8deg. PERTURB.IN PITCH FROM EQUILIBRIUM



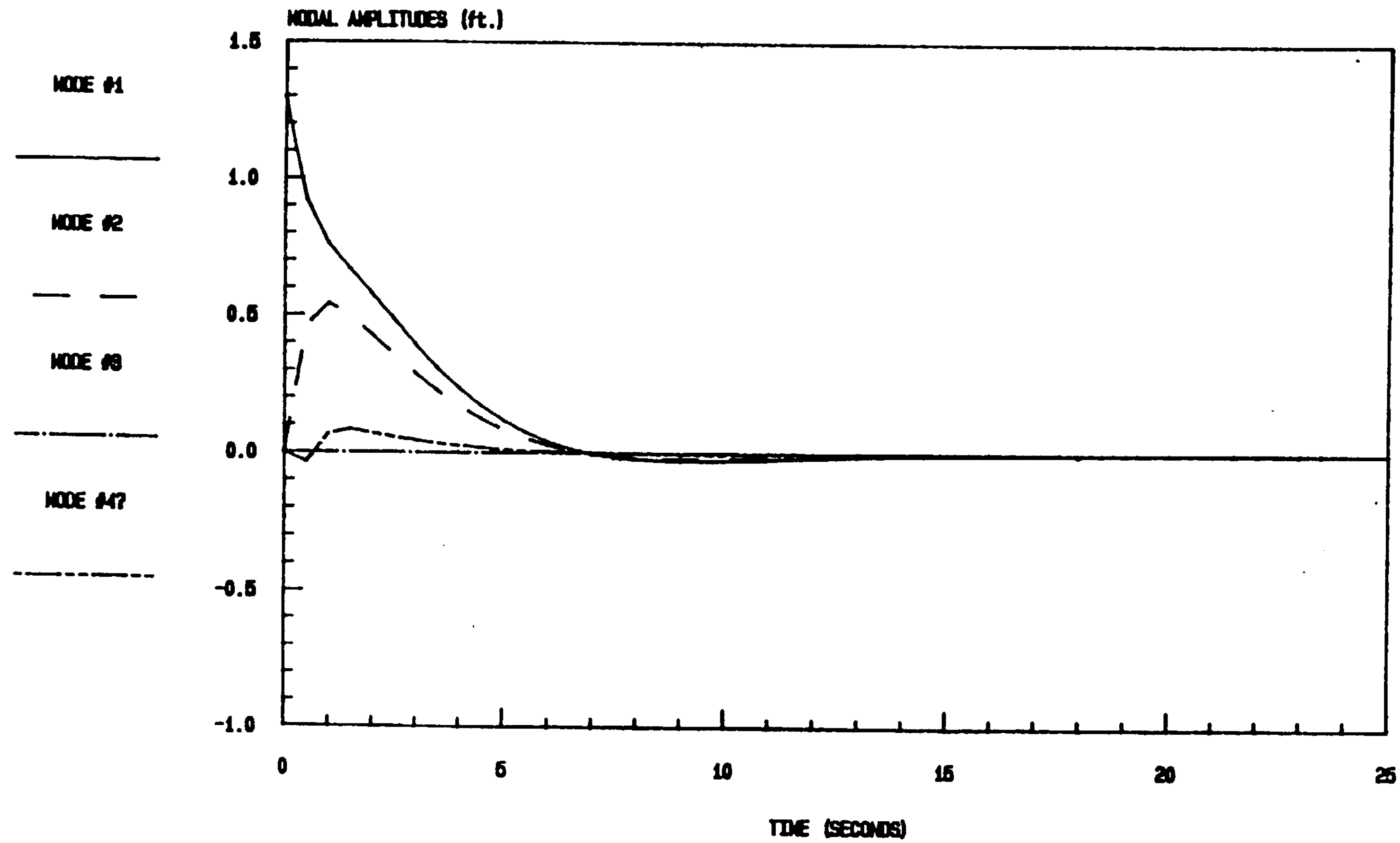
LINEAR MODEL OF SCOLE WITH FLEXIBILITY

TRANS. RESP. TO A 1.8ft.DIST.IN MODE #1



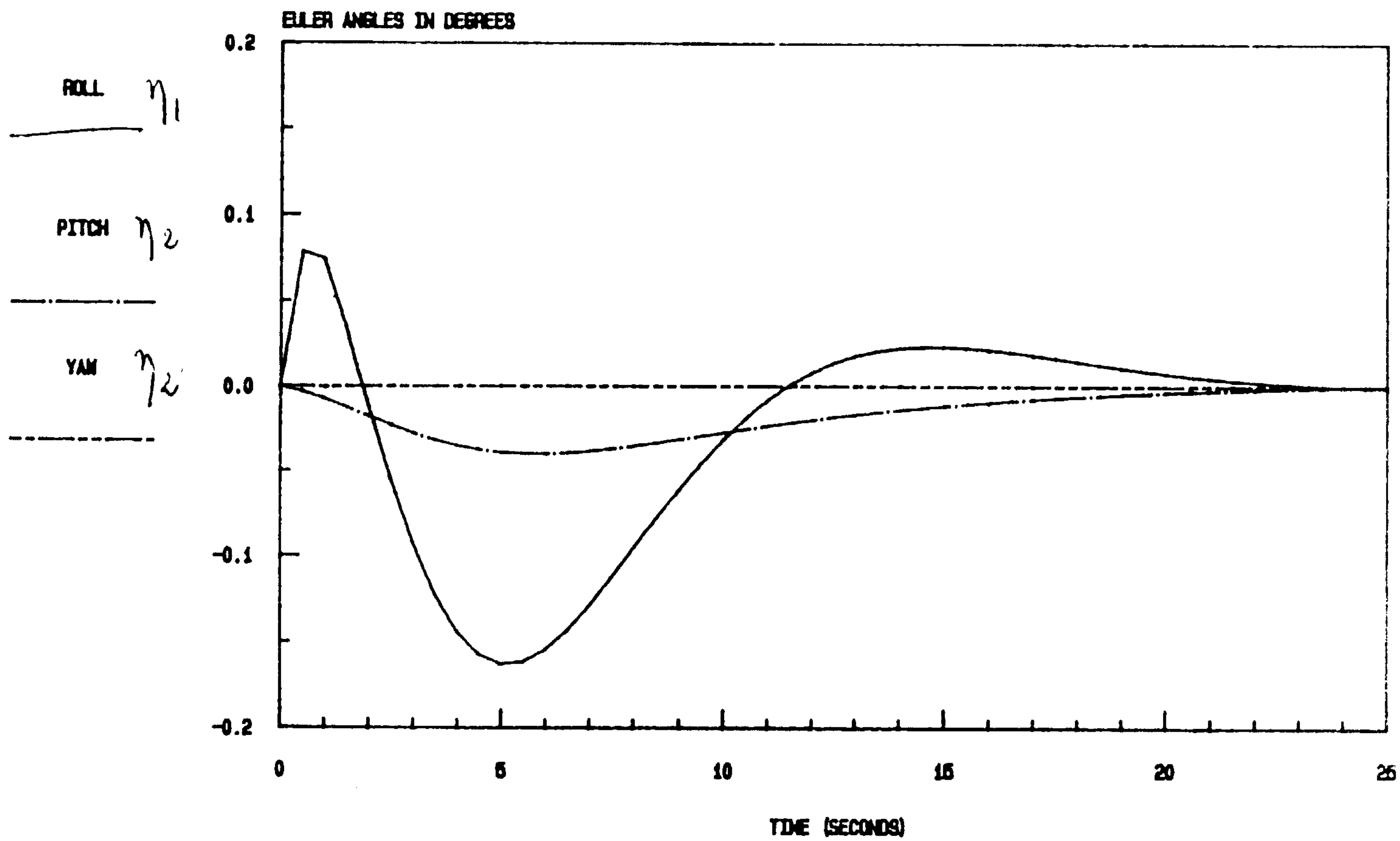
LINEAR MODEL OF SCOLE WITH FLEXIBILITY

TRANS. RESP. TO A 1.8ft.DIST.IN MODE #1



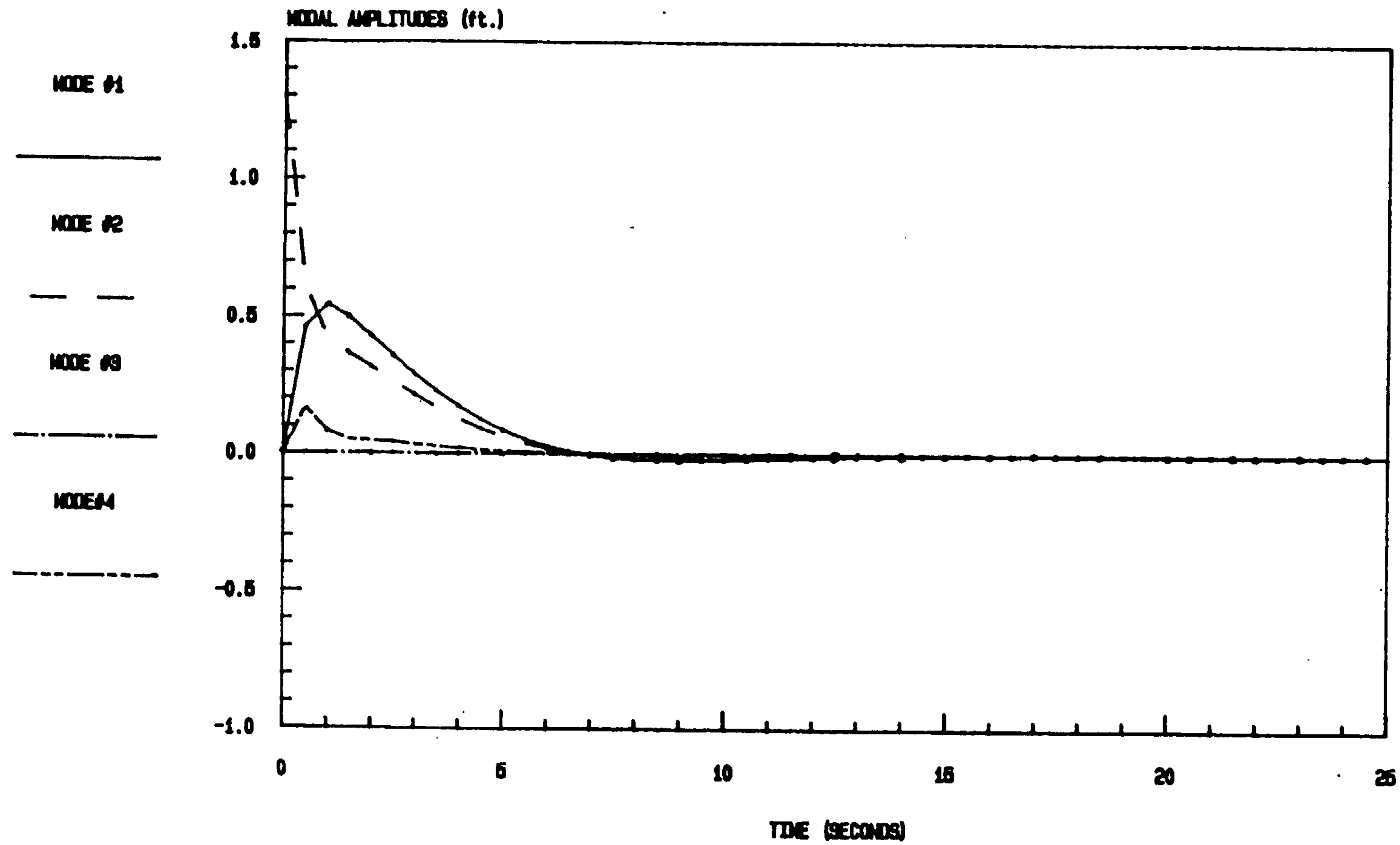
LINEAR MODEl OF SCOLE WITH FLEXIBILITY

TRANS. RESP. TO A 1.8ft.DIST.IN MODE #2



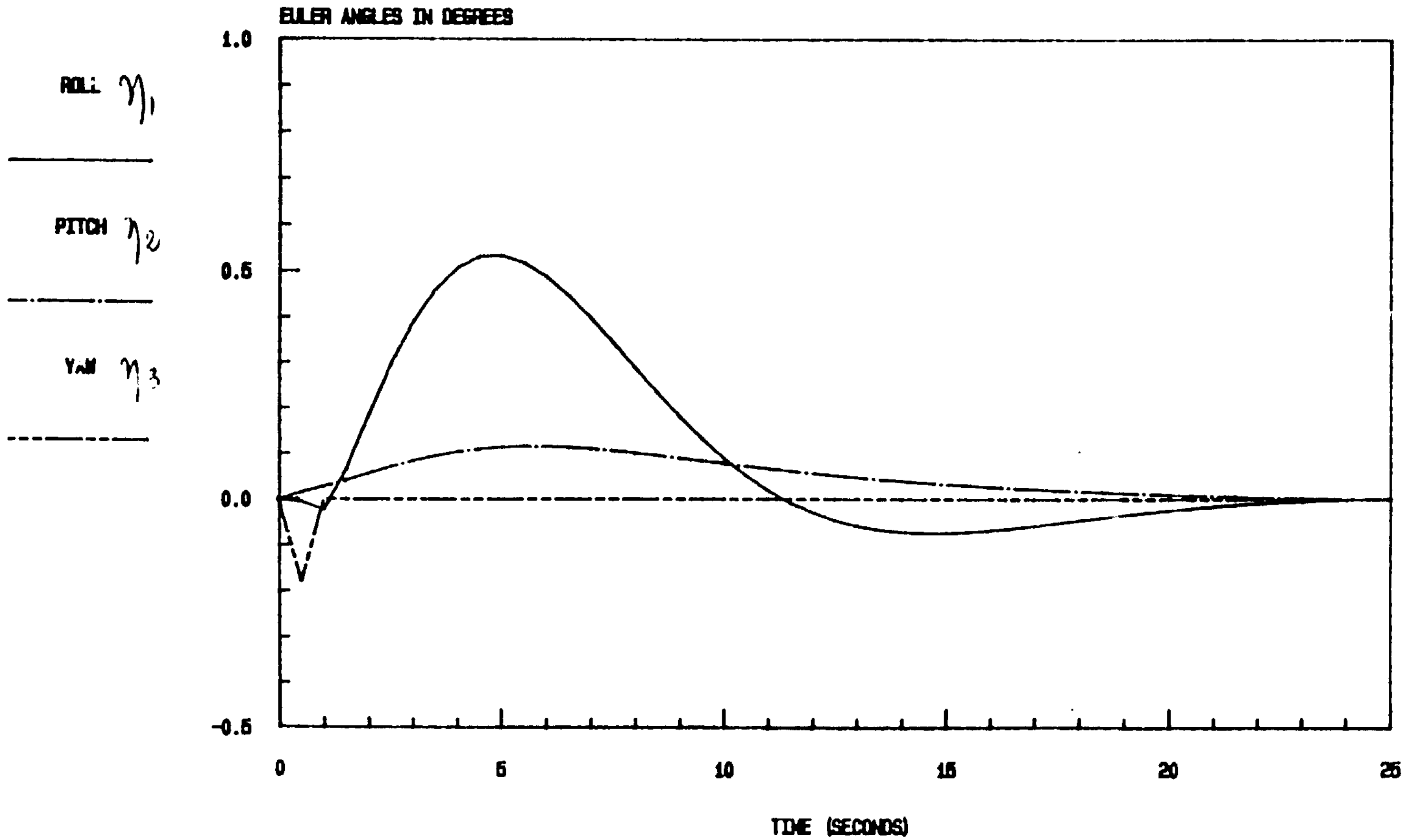
LINEAR MODEL OF SCOLE WITH FLEXIBILITY

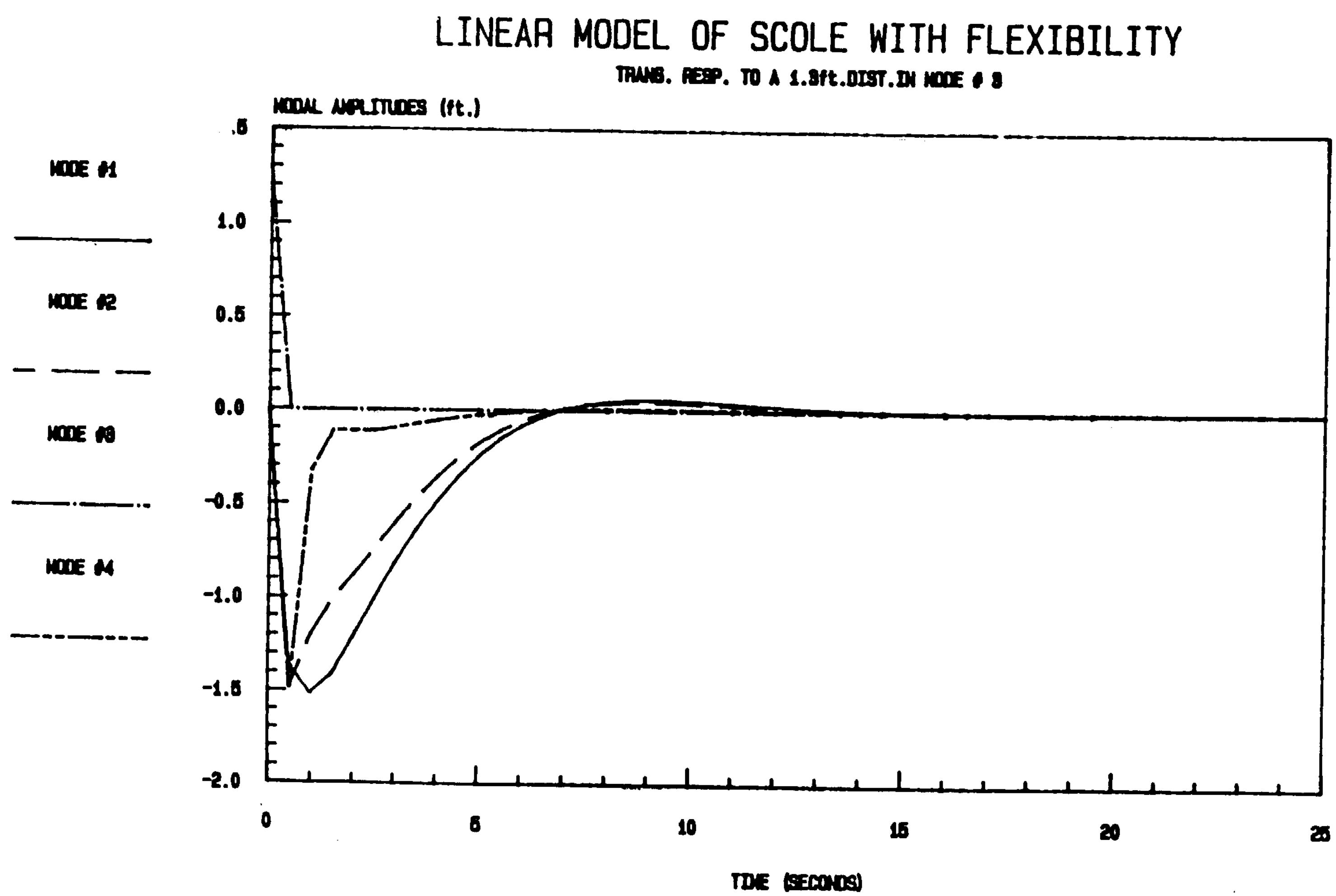
TRANS. RESP. TO A 1.8ft.DIST.IN MODE #2



LINEAR MODEL OF SCOLE WITH FLEXIBILITY

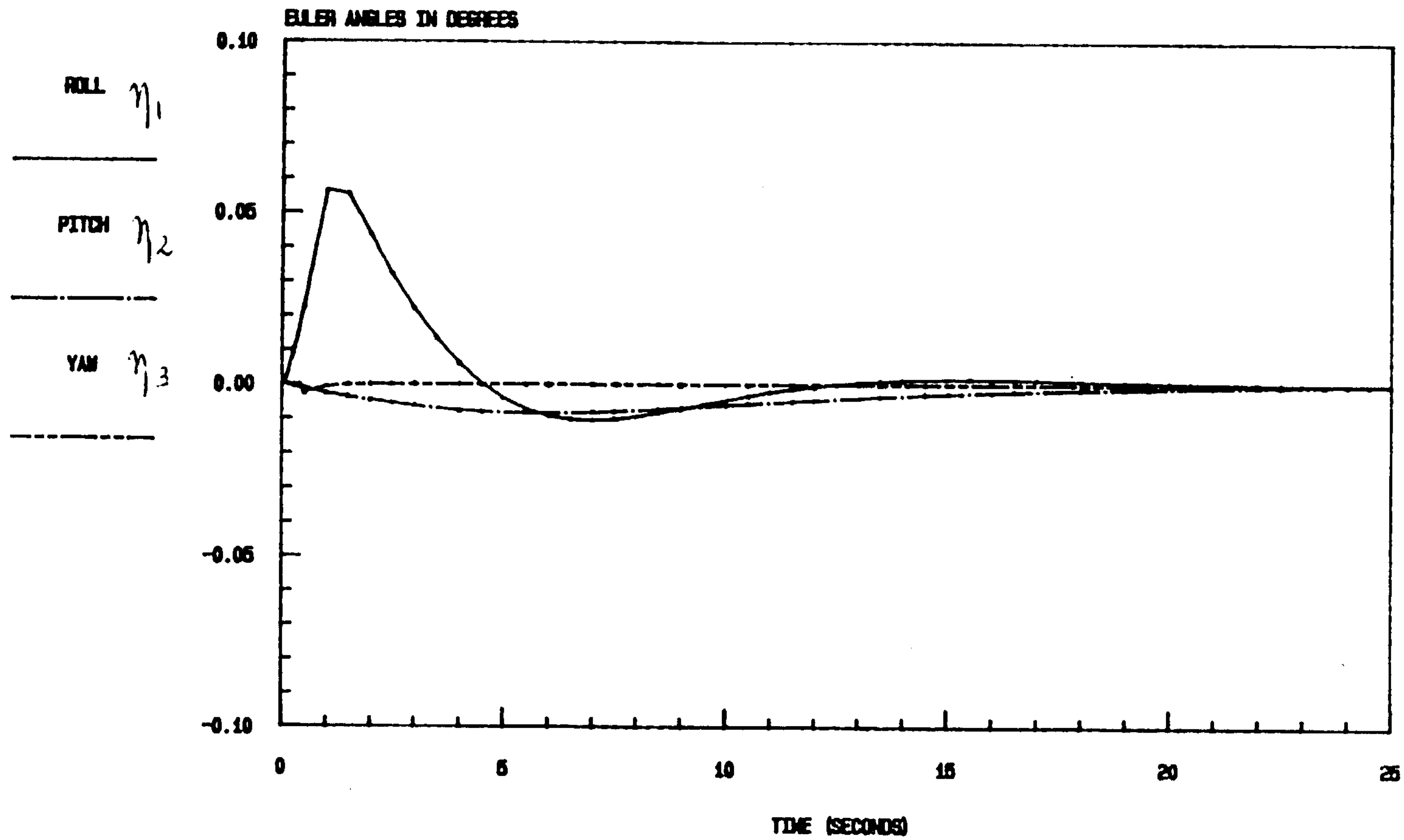
TRANS. RESP. TO A 1.8ft.DIST.IN MODE #3




LINEAR MODEL OF SCOLE WITH FLEXIBILITY
TRANS. RESP. TO A 1.0ft.DIST.IN MODE # 3
MODAL AMPLITUDES (ft.)
MODE #1
MODE #2
MODE #3
MODE #4
.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
0
5
10
15
20
25
TIME (SECONDS)

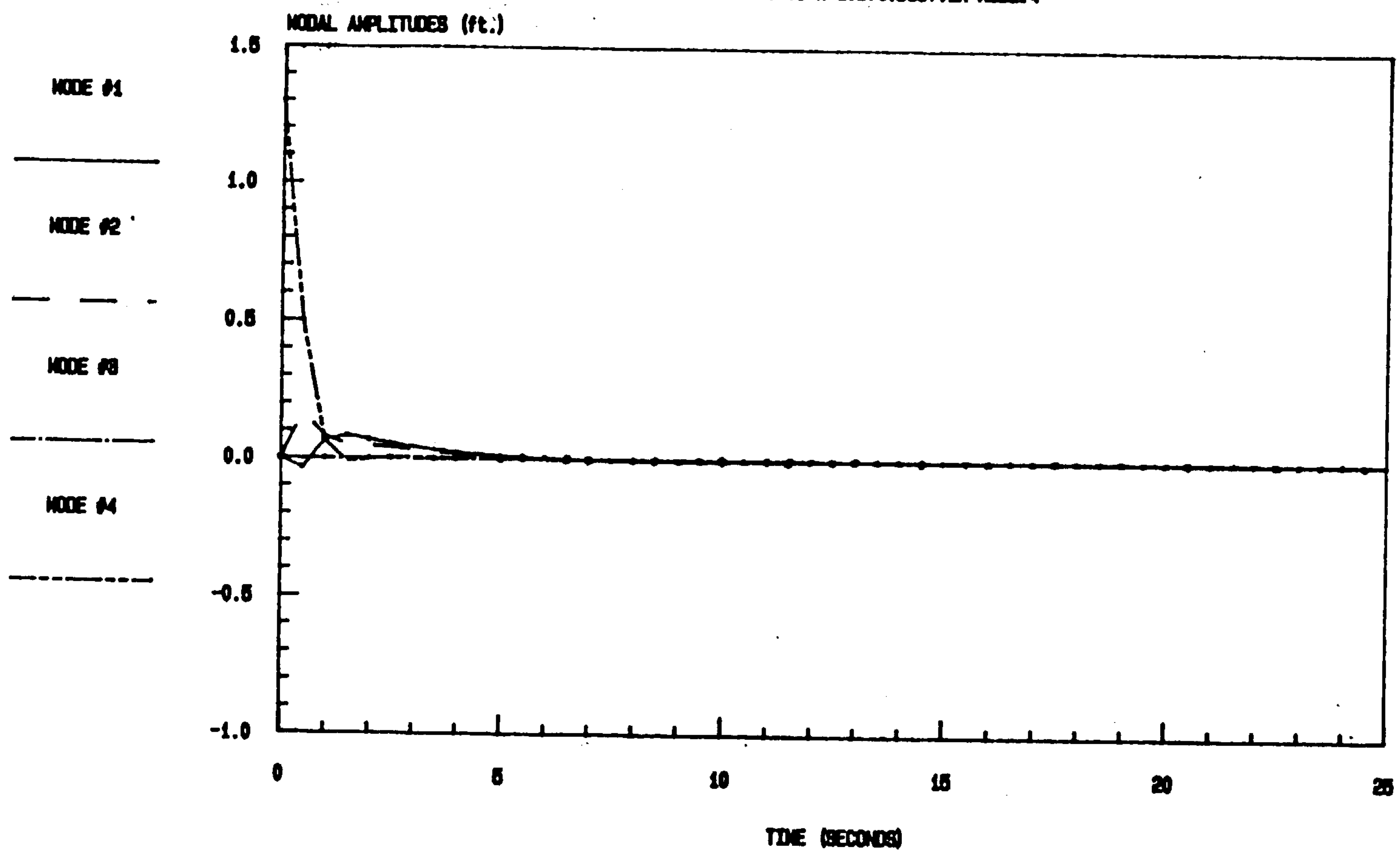
LINEAR MODEL OF SCOLE WITH FLEXIBILITY

TRANS.RESP. TO A 1.8ft.DIST.IN MODE #4



LINEAR MODEL OF SCOLE WITH FLEXIBILITY·

TRANS. RESP. TO A 1.8ft.DIST.IN MODE#4



MINIMUM TIME ATTITUDE SLEWING
MANEUVERS OF A RIGID SPACECRAFT

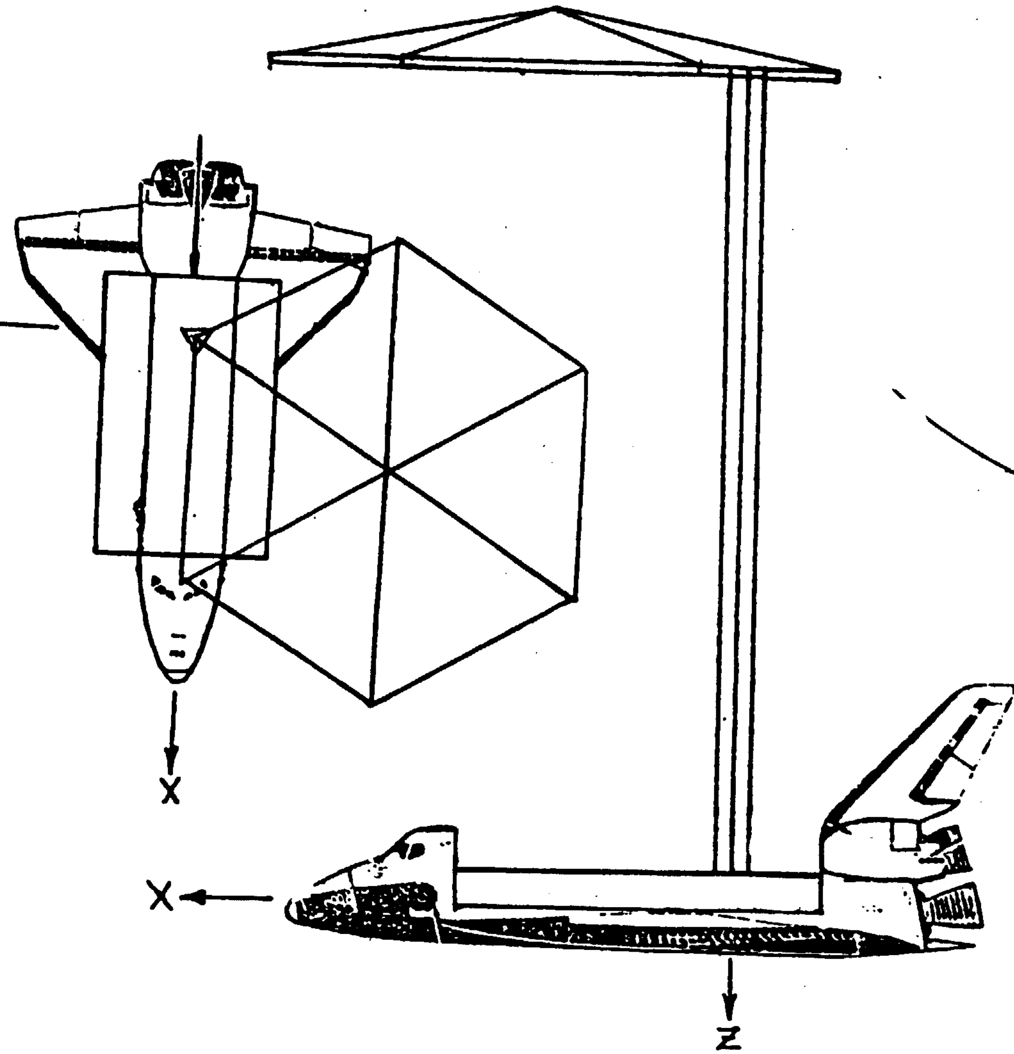
OBJECTIVE

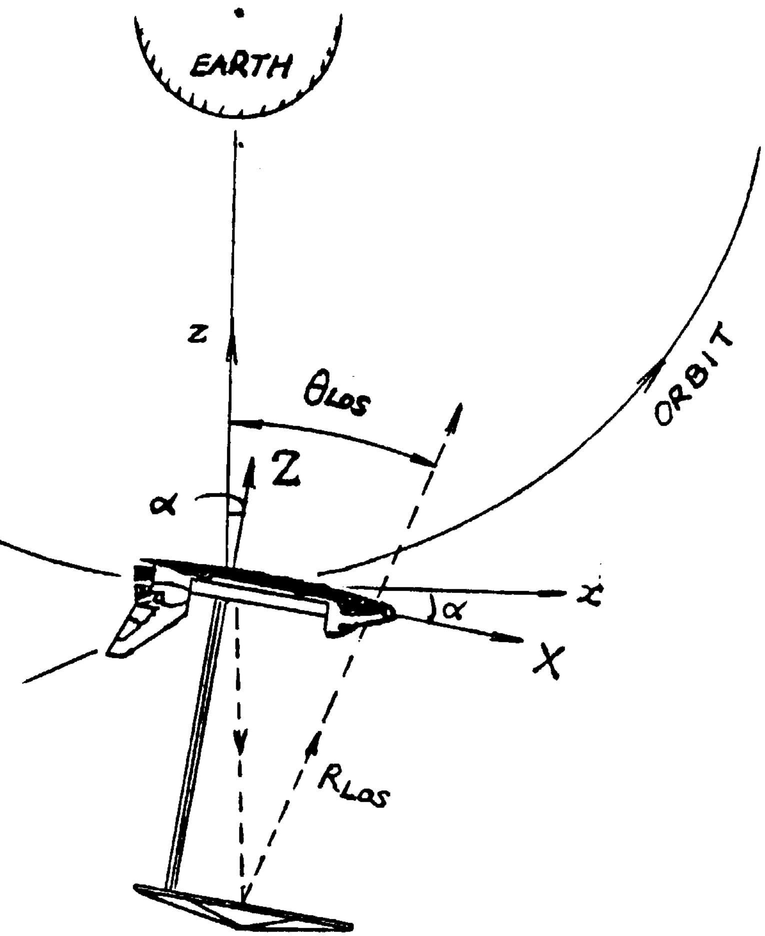
- DEVELOP COMPUTATIONAL TECHNIQUES TO SLEW A GENERAL RIGID SPACECRAFT (INCLUDING RIGIDIZED SCOLE) FROM AN ARBITRARY INITIAL ATTITUDE TO A FINAL REQUIRED ATTITUDE PRECISELY, AND SATISFYING THE FOLLOWING CONDITIONS:
    - IN MINIMUM TIME
    - THE CONTROLS HAVE SATURATION LEVELS

METHODOLOGY

- THE MAXIMUM PRINCIPLE FROM OPTIMAL CONTROL THEORY IS APPLIED TO THE EULER'S DYNAMICAL EQUATIONS AND THE QUATERNION KINEMATICAL EQUATIONS OF THE SYSTEM TO DERIVE THE NECESSARY CONDITIONS FOR THE CONTROLS. THIS LEADS TO THE TWO-POINT BOUNDARY-VALUE PROBLEM.
- AN INTEGRAL OF A QUADRATIC FUNCTION OF THE CONTROLS IS USED AS A COST FUNCTION, BUT THE INTEGRATION PERIOD OF THIS INTEGRAL, CALLED THE SLEWING TIME, IS TO BE CHANGED UNTIL IT REACHES ITS MINIMUM VALUE.
- THE RESULTING TPBVP IS SOLVED BY A QUASILINEARIZATION ALGORITHM (METHOD OF PARTICULAR SOLUTIONS).
- EULER'S EIGENAXIS ROTATION THEOREM IS USED TO APPROXIMATELY DETERMINE THE INITIAL VALUES OF THE COSTATES AND THE SLEWING TIME AS WELL AS THE NOMINAL SOLUTIONS WHICH ARE UESD TO START THE QUASILINEARIZATION ALGORITHM.
- THE MINIMUM SLEWING TIME IS DETERMINED BY SHORTENING THE TOTAL SLEWING TIME UNTIL AT LEAST ONE OF THE CONTROLS BECOMES A BANG-BANG TYPE.

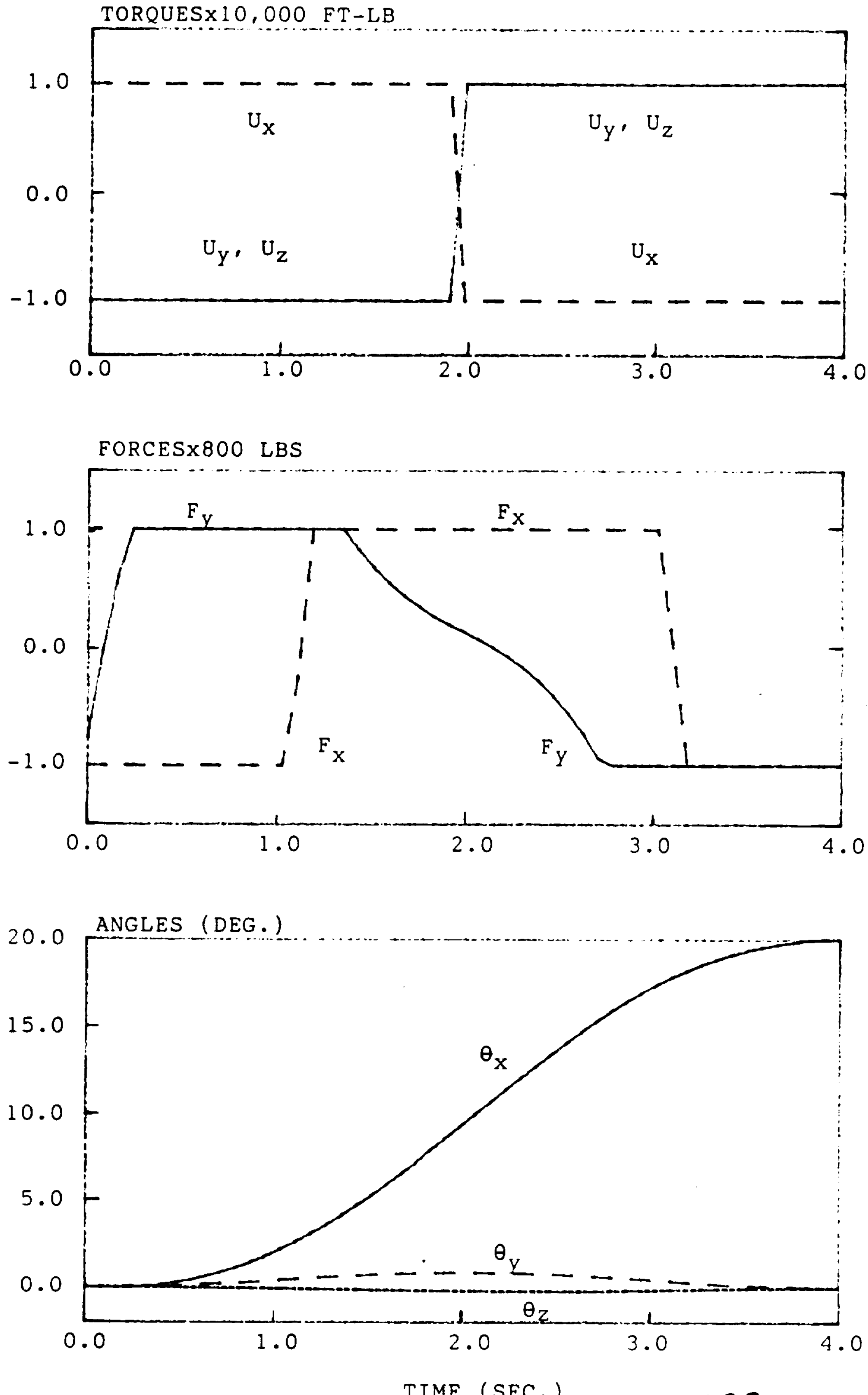
SPACECRAFT CONTROL LAB EXPERIMENT
(SCOLE)



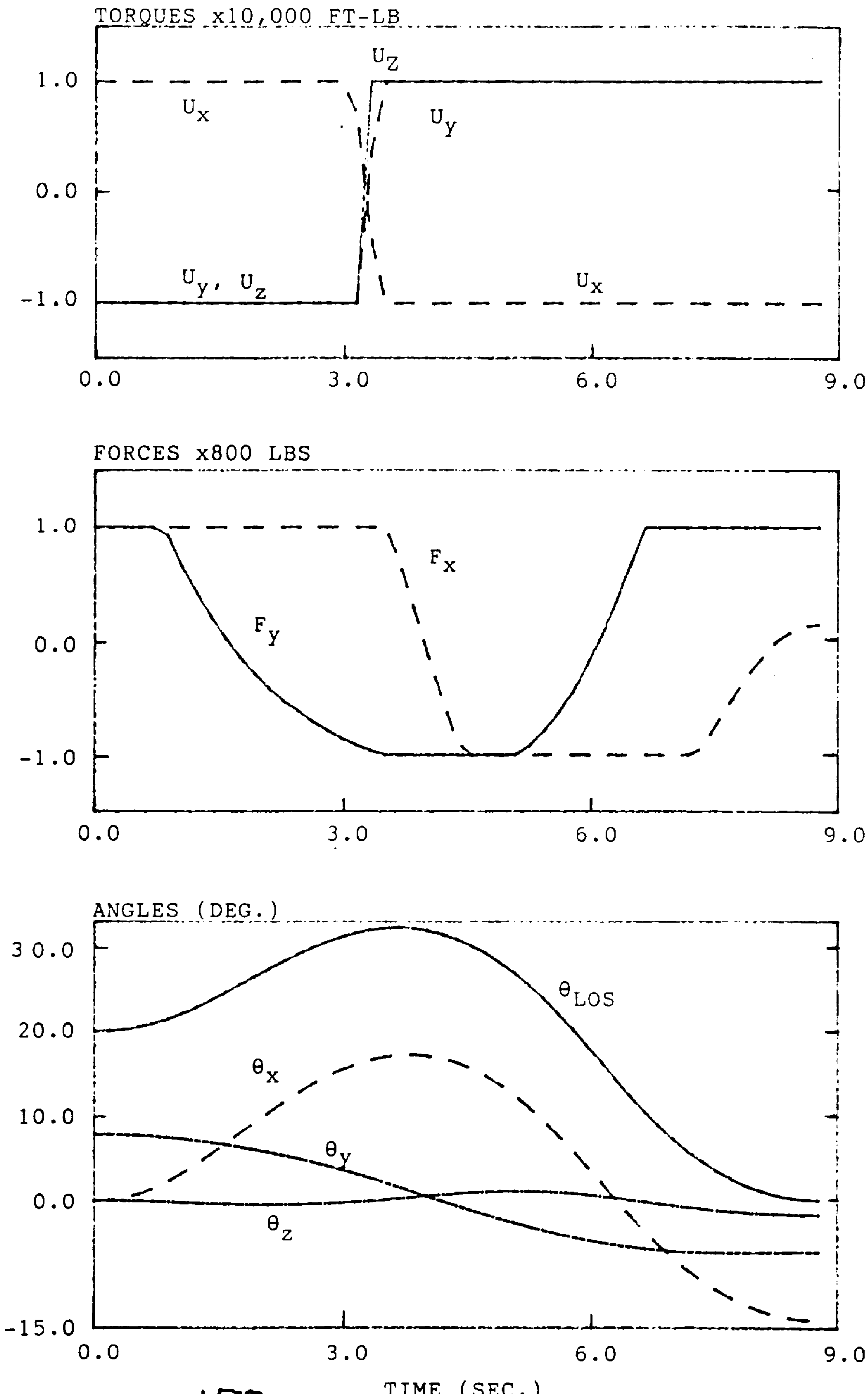


ATTITUDE OF THE SCOLE

X-AXIS SLEWING (TIME = 3.888 SEC.)



EXAMPLE-SLEWING (TIME = 8.77 SEC.)



CONCLUDING REMARKS

- THE SLEWING MOTION NEED NOT BE RESTRICTED TO A SINGLE AXIS MANEUVER.
- THE GUESSED STARTING VALUE OF THE SLEWING TIME IS VERY CLOSE TO THE CONVERGED VALUE FOR THE SCOLE EXAMPLES AND SUBROUTINE USED HERE.
- THE GUESSED INITIAL VALUES OF THE COSTATES ARE ADEQUATE FOR THE ALGORITHM TO CONVERGE.
- THE METHODS USED HERE MAY BE IMPLEMENTED FOR PRACTICAL CONTROL SOURCES WHICH MAY HAVE MORE CONSTRAINTS.
- AN EXTENSION TO THE MINIMUM TIME SLEWING MOTION OF THE SCOLE MODEL CONTAINING BOTH RIGID AND FLEXIBLE COMPONENTS IS PLANNED.

Appendix - Chapter II

Stability Analysis of Second Order System with Delayed State Feedback

As a second order differential equation describes the dynamics of a single mode of any large space structure, the stability analysis of such a system with delayed state feedback is analyzed and the amount of delay that can be tolerated by the system without becoming unstable is arrived at analytically.

The differential equation of second order with state feedback can be written as:

$$\ddot{x}_i + 2\zeta_i'\omega_i\dot{x}_i + \omega_i^2 x_i = -k_r x_i(t-h) - k_p\dot{x}_i(t-h) \tag{1}$$

where

x_i = i^{th} modal coordinate

ω_i = i^{th} natural frequency

ζ_i' = i^{th} mode inherent damping ratio

k_r = rate feedback gain

k_p = position feedback gain

h = time delay

The feedback gains k_r, k_p are designed for the required stability and transient response specifications without taking the delay into consideration.

The inherent damping ratio, ζ_i' and the feedback gains, k_r and k_p; will give rise to five possible combinations as shown in Table 1 and are thus analyzed separately for mathematical convenience and easy understanding.

Case I: $\zeta_i' = 0$, $k_p = 0$ and $k_r > 0$

The differential equation of the system can be written as:

$$\ddot{x}_i + \omega_i^2 x_i = - k_r \dot{x}_i(t-h) \tag{3}$$

Case	ζ_i'	k_r	k_p
I	= 0	> 0	= 0
II	> 0	> 0	= 0
III	= 0	> 0	> 0
IV	> 0	= 0	≠ 0
V	> 0	> 0	≠ 0

Note: The remaining three combinations are neither feasible nor of interest.

Table 1: Feasible Combinations of ζ_i', k_r, k_p for Stability Analysis

and the corresponding characterstic equation is given by:

$$s^2 + \omega_i^2 + 2\zeta_i\omega_i s e^{-sh} = 0 \tag{4a}$$

where $k_r = 2\zeta_i\omega_i$.

The value of h for which the roots of equation (3) cross the imaginary axis can be evaluated by substituting $s = j\omega$.

Thus

$$\omega_i^2 - \omega^2 + j2\zeta_i\omega_i\omega\sin\omega h + 2\zeta_i\omega_i\omega\cos\omega h = 0 \tag{4b}$$

For equation (4b) to be satisfied

$$\sin\omega h = 0$$

and $$\omega_i^2 - \omega^2 + 2\zeta_i\omega_i\omega \cos\omega h = 0 \tag{5}$$

Thus $\omega h = \pi/2$

and
$$h = \frac{\pi/2}{\omega_i\,[\zeta_i + \sqrt{1+\zeta_i^2}\,]} \tag{6}$$

Case II: $\zeta_i' > 0,\ k_r = 2\zeta_i\omega_i$ and $k_p = 0$

The characteristic equation of the system described by equation (1) is given by

$$(\omega_i^2-\omega^2+2\zeta_i\omega_i\omega\sin\omega h) + j(2\zeta_i'\omega_i\omega+2\zeta_i\omega_i\omega\cos\omega h) = 0 \tag{7}$$

Thus $\cos\omega h = -\zeta_i'/\zeta_i$

and
$$h = \frac{\cos^{-1}(\zeta_i'/\zeta_i}{\omega_i\,[\sqrt{\zeta_i^2-\zeta_i'^2} + \sqrt{1+\zeta_i^2-\zeta_i'^2}]\,]} \tag{8}$$

For the case where $\zeta_i<\zeta_i'$ thesystem will always be stable since no value of h exists for which the roots of (7) cross the imaginary axis. A plot of $\omega_i h$ versus ζ_i for various values of ζ_i' is shown in Figure 2.1.

Case III: $\zeta_i' = 0,\ k_p = k_r > 0$

The characteristic equation is given by

$$s^2+\omega_i^2+k_r s e^{-sh}+k_p e^{-sh} = 0 \tag{9}$$

or
$$(\omega_i^2-\omega_i^2+\omega k_r\sin\omega h+k_p\cos\omega h) + j(\omega k_r\cos\omega h-k_p\sin\omega h) = 0 \tag{10}$$

Thus
$$\tan\omega h = \frac{\omega k_r}{k_p}$$

and
$$\omega^2 = \frac{1}{2}\,[(2\omega_i^2+k_r^2) + \sqrt{[k_r^4+4\omega_i^2k_r^2+4k_p^2]}] \tag{11}$$

Plots of $h\omega_i$ versus k_r/ω_i for various values of k_p/ω_i^2 are shown in Figure 2.2. It can be seen here that these are many combinations of k_p and k_r for which the roots of Eq. (10) can cross the imaginary axis - i.e. value of $h\omega_i$ which leads to instability.

Case IV: $\zeta_i' > 0,\ k_r = 0,\ k_p \neq 0$

The characteristic equation is given by

$$(\omega_i^2 - \omega^2 + k_p \cos\omega h) + j(2\zeta_i'\omega_i\omega - k_p \sin\omega h) = 0 \qquad (12)$$

Thus

$$\sin\omega h = \frac{2\zeta_i'\omega_i\omega}{k_p} \qquad (13)$$

and

$$\omega^2 = \omega_i^2(1-2\zeta_i'^2) + \omega_i^2\sqrt{[(1-2\zeta_i'^2)^2 + (k_p/\omega_i^2)^2]} \qquad (14)$$

The plots of $h\omega_i$ versus k_p/ω_i^2 for various values of ζ_i' are shown in Figure 2.3

Case V: $\zeta_i' > 0,\ k_r > 0,\ k_p \neq 0$

The characteristic equation is given by

$$(\omega_i^2 - \omega^2 + \omega k_r \sin\omega h + k_p \cos\omega h) + j(2\zeta_i'\omega_i\omega + \omega k_r \cos\omega h - k_p \sin\omega h) = 0 \qquad (15)$$

By equating the imaginary part to zero, ωh can be evaluated as

$$\omega h = \sin^{-1}\Bigg(\underbrace{\frac{2\zeta_i'\omega_i\omega}{\sqrt{k_p^2 + \omega k_r^2}}}_{y}\Bigg) - \tan^{-1}\left(\frac{\omega k_r}{k_p}\right) \qquad (16)$$

after substituting ωh in the real part of equation (15), the following equation in the single unknown variable ω can be obtained

$$\omega_i^2 - \omega^2 + \omega k_r \sin\left(\sin^{-1} y - \tan^{-1}\left(\frac{\omega k_r}{k_p}\right)\right) + k_p \cos\left(\sin^{-1} y - \tan^{-1}\left(\frac{\omega k_r}{k_p}\right)\right) = 0 \qquad (17)$$

Using equations (17) and (16), the limiting value for given values of ζ_i', k_r, k_p and ω_i can be determined. As the equation (17) is nonlinear, numerical procedures may have to be used and thus the generalized plots similar to the other cases may be obtained.

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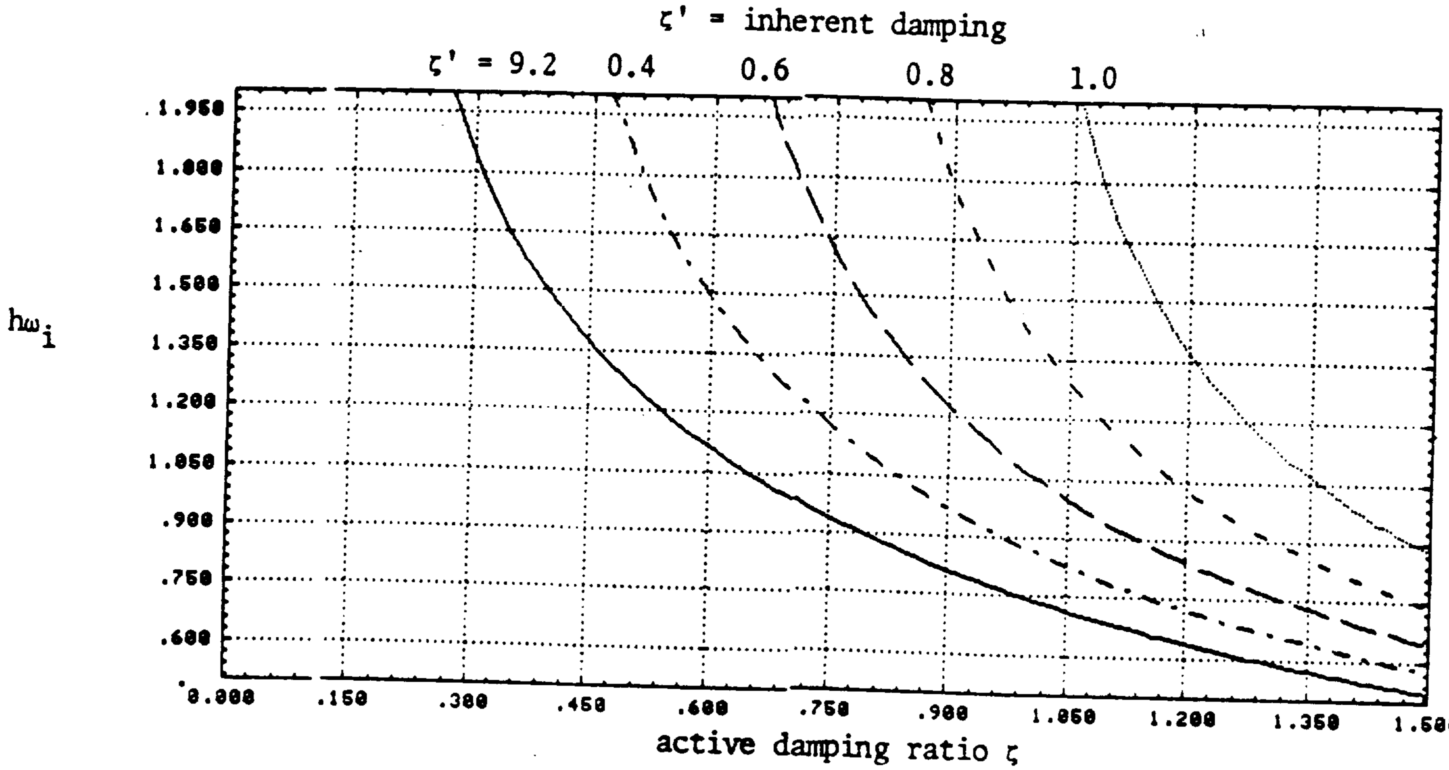

Figure 2.1: Plots of $h\omega_i$ vs ζ_i correspondence to Case II with ζ_i' as a parameter.

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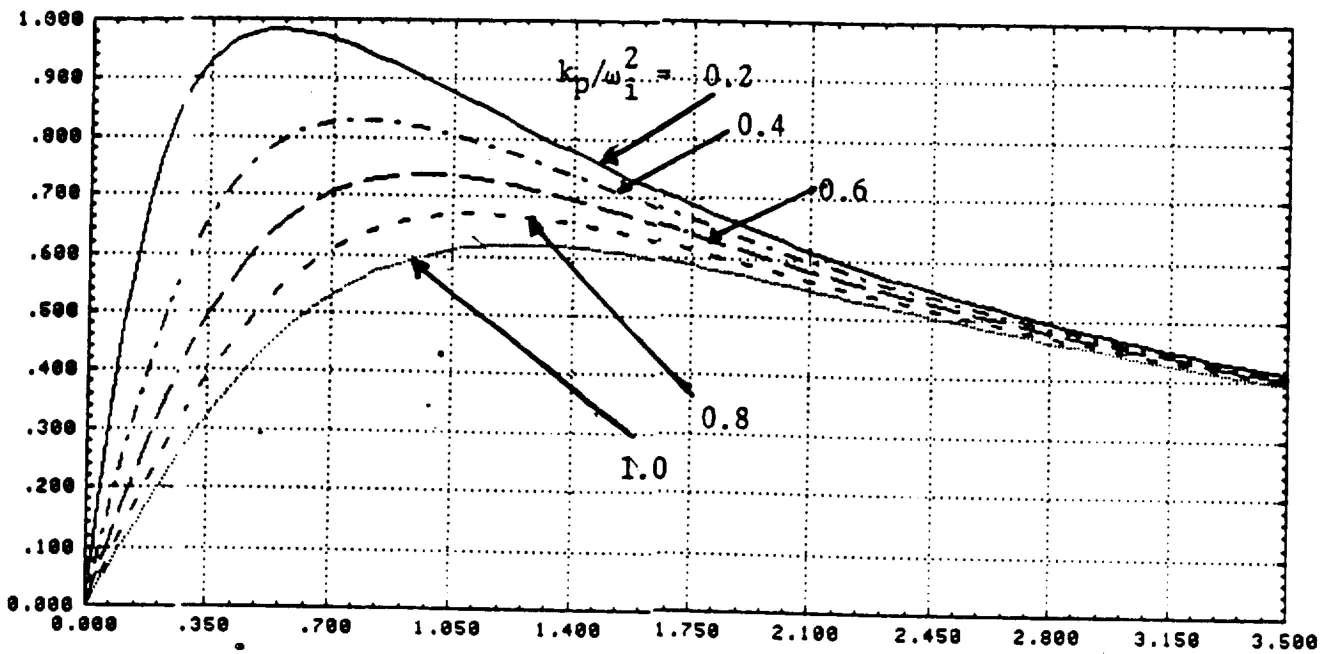

Figure 2.2 Plot of $h\omega_i$ vs k_r/ω_i corresponding to Case III with k_p/ω_i^2 as a parameter

178

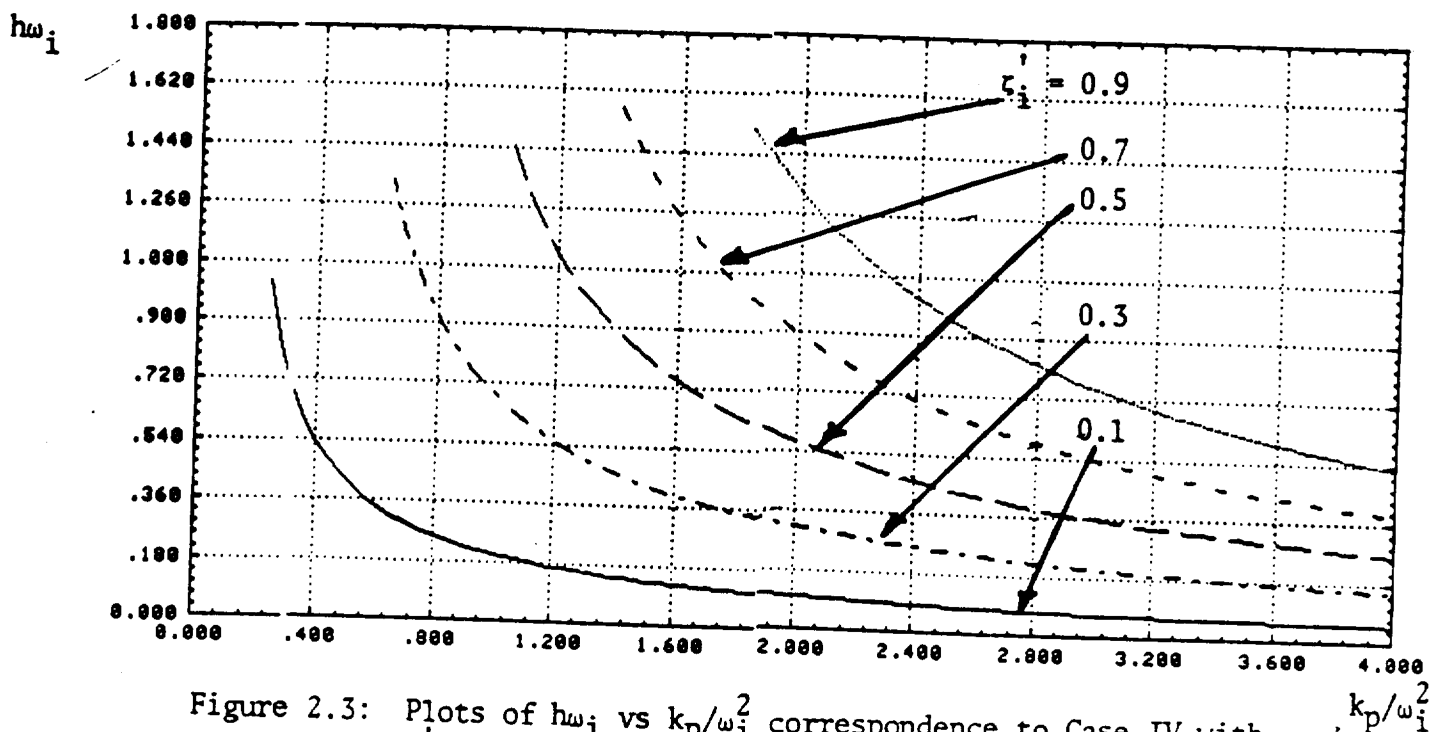

Figure 2.3: Plots of $h\omega_i$ vs k_p/ω_i^2 correspondence to Case IV with ζ_i' as a parameter

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$h\omega_i$